Analysis of 13 years (1993-2005) of Satellite Laser Ranging data on the two LAGEOS satellites for Terrestrial Reference Frames and Earth Orientation Parameters

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Abstract

The quality presently reached by space-geodetic techniques, regarding precision, accuracy such as spatial and temporal distributions of their measurements, allows us to compute time series of geodetic products.

In this context, we have developed a method to compute time series of Earth Orientation Parameters (EOPs) and terrestrial station positions through the analysis of Satellite Laser Ranging (SLR) data. This technique being an important basis for the computation of the International Terrestrial Reference Frame, it is crucial to derive accurate time series with a rigorous approach. Furthermore, this method will be used by the scientific department GEMINI of the Observatoire de la Côte d'Azur when it will become an official ILRS analysis center.

These times series are obtained with a good accuracy and a reasonable sampling (1 day for EOPs and 1 week for station positions). This good accuracy is ensured by i) a rigorous weighting of SLR measurements per satellite and per station; ii) a kinematic approach to compute orbital residual errors; iii) a rigorous control of range biases which is detailed in [Coulot et al.,2007].

In this paper, we first present the two aspects i) and ii) of our method. In a second part, we analyze 13 years (1993-2005) of SLR data on both LAGEOS satellites in order to study the Terrestrial Reference Frames and the EOPs so computed.

Introduction

This paper comprises four parts. First, we detail the two LAGEOS satellite orbit computation. Second, we provide general considerations about the Satellite Laser Ranging (SLR) data processing, regarding the data weighting, the orbital residual errors, and the range biases. Then, we describe the time series computation method and produce the results and, finally, we provide some conclusions and prospects.

1. Orbit computation

This section aims to briefly describe the two LAGEOS satellite orbit computation. Tables 1, 2, and 3 respectively show the physical models used for the orbit computations and for the Earth Orientation Parameters (EOPs) and the station positions during these computations.

Fig.1 shows the orbit residual WRMS and the numbers of data used and rejected for both satellites. Tab. 4 provides some statistics of these values. We can see that, on average, the residual WRMS are at the centimeter level for both LAGEOS satellites.

The sampling used for these computations is the GPS week but, in order to reduce the impact of the residual orbital errors, we in fact compute 9-day orbital arcs and only keep the 7-day central arcs. As a result, our orbital arcs provide 2-day overlaps. Fig. 2

shows the bias and the RMS values of the orbit differences so computed in RTN frame for both satellites. Table 5 provides the mean values of these difference bias and RMS values.

Table 1. *Physical models used for the orbit computations.*

Type	Description
Earth's gravity field	GRIM5_C1 [Gruber et al., 2000]
Atmospheric density	DTM94 [Berger et al., 1998]
Planetary ephemerides	DE403 [Standish et al., 1995]
Earth's time varying gravity field	
Solid Earth tides	Model in [McCarthy and Petit, 2004]
Solid Earth pole tide	Model in [McCarthy and Petit, 2004]
Oceanic tides	FES2002 [Le Provost, 2002]
Atmospheric pressure	ECMWF, http://www.ecmwf.int/

Table 2. *Physical models used for the EOPs during the orbit computations.*

Type	Description
Reference time series	EOPC04 [Gambis, 2004]
Quasi-diurnal Variations	Model in [McCarthy and Petit, 2004]
Precession	Model [Lieske et al., 1977]
Nutation	Model in [McCarthy, 1996]

Table 3. *Physical models used for the stations positions during the orbit computations.*

Type	Description
Terrestrial Reference Frame	ITRF2000 [Altamimi et al., 2002]
Celestial Reference Frame	ICRF [Arias et al., 1995]
Solid Earth tides	Model in [McCarthy and Petit, 2004]
Solid Earth pole tide	Model in [McCarthy and Petit, 2004]
Oceanic loading (only tidal components)	Computed with FES2002
Atmospheric loading (only non-tidal components)	Computed with ECMWF fields

Table 4. *Statistics of the values shown on Fig. 1.*

Satellite	Mean residual WRMS	Mean number of data used	Mean number of rejected data
LAGEOS	1.11 cm	1433	49
LAGEOS-2	0.95 cm	1320	35

Their interpretation is not easy, and yet these overlaps provide a way of controlling the orbit quality. From Table 5, we can see that the two LAGEOS satellite orbits provide differences with mean RMS values between 1 and 4.5 cm.

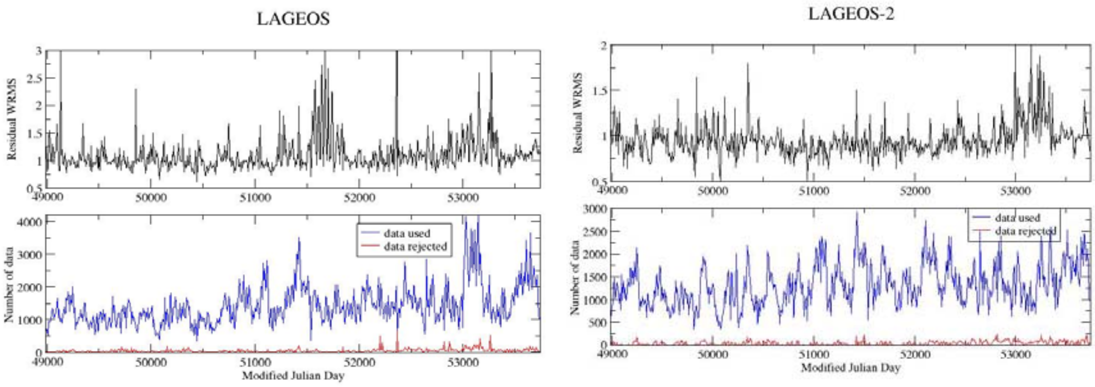

Figure 1.*Orbit residual WRMS (cm) (black curves) and numbers of data used (blue curves) and rejected (red curves) per orbital arc for both LAGEOS satellites (LAGEOS on the left and LAGEOS-2 on the right).*

Table 5. *Statistics of the values shown on Fig. 2.*

Satellite	R (cm)	T (cm)	N (cm)	.
LAGEOS	-0.02 2.57	-0.01 4.37	0.01 2.59	Mean bias Mean RMS
LAGEOS-2	0.01 1.32	-0.05 2.26	0.00 2.66	. .

2. General considerations

The SLR data processing method we have developed is divided in three steps. Fig. 3 shows the global computational scheme. First, GRGS (french Groupe de Recherche en Géodésie Spatiale, Spatial Geodesy Research Group, in English) GINS (Géodésie par Intégration Numérique Simultanée, Geodesy by Simultaneous Numerical Integration, in English) software provides the two LAGEOS satellite orbits with the help of physical models and SLR measurements (see previous section 1). Second, GRGS MATLO (MAThématiques pour la Localisation et l'Orbitographie, MAThematics for Localization and Orbitography, in English) software uses these orbital arcs and the SLR data to compute pseudo measurements as well as partial derivatives of these latter with respect to the parameters worthy of interest. Finally, an estimation software (POSGLOB for POSitionnement GLOBal or GLOBal POSitioning in English) produces parameter estimates from MATLO outputs.

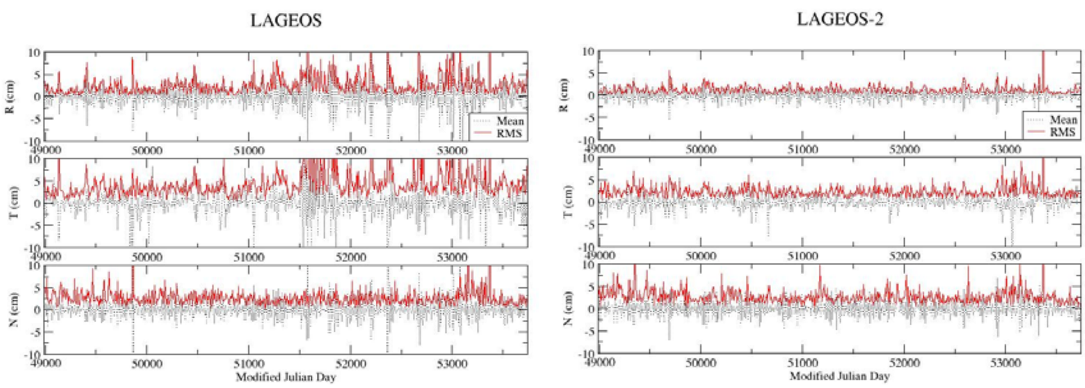

Figure 2. *Orbit differences (biases - in black - and RMS values - in red -, in cm) in the RTN frame computed over the two overlapping days for both LAGEOS satellites (LAGEOS on the left and LAGEOS-2 on the right).*

As shown in green boxes on Fig. 3, there are three critical issues in such computation: the range bias and residual orbital error handling and the data weighting. Thus, we try to build the optimal method to take these issues into account.

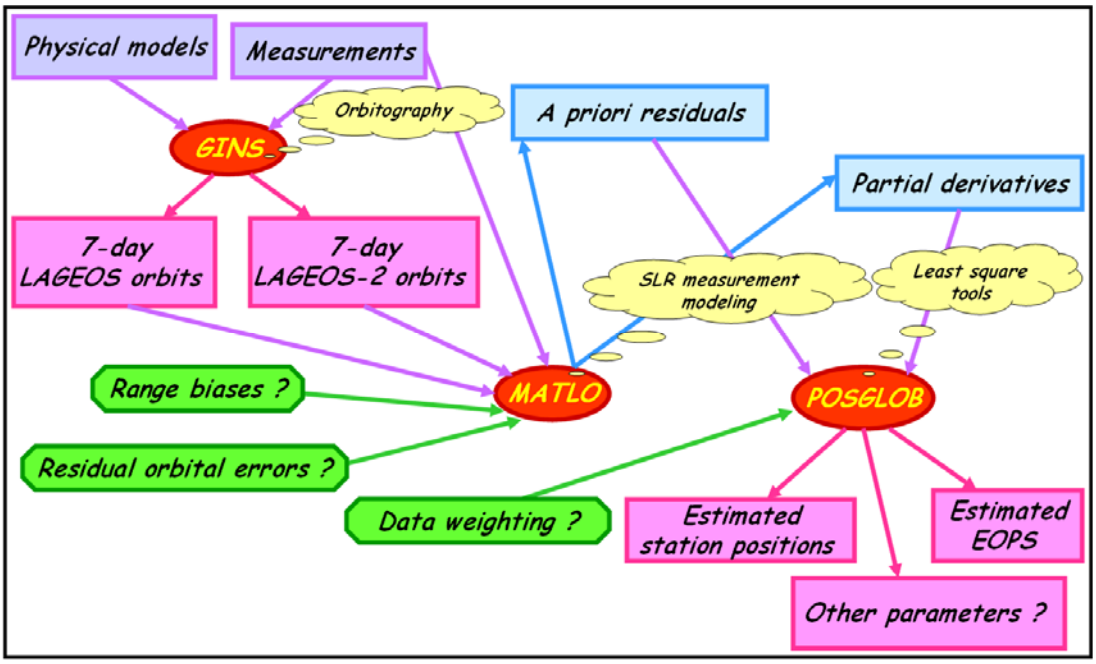

Figure 3. *SLR data processing scheme.*

2.1. Data weighting

SLR stations do not provide measurements of the same quality. As a consequence, we can not use the same weight for all SLR measurements but we have to find weights which really correspond to the quality of these measurements. To do so, we use an optimal variance component analysis method: the degree of freedom method inspired by [Persson, 1982]. The following scheme on Fig. 4 summarizes the method (see [Sillard, 1999] and [Coulot, 2005] for more details).

As shown on Fig.4, this method (as a great part of such variance component analysis method) is based on common parameters for all considered observation groups. In our case, the only real common parameters are EOPs as we consider that observation groups are measurements per station and per satellite. Thus, our variance component analysis approach only relies on these EOPs.

Fig. 5 shows the method used to derive the optimal weighting per station and per satellite. First of all, MATLO software is used to derive pseudo measurements and partial derivatives of these latter with respect to station positions and EOPs from the 7-day LAGEOS satellite orbits and the range biases computed with the temporal de-correlation method (see section 2.3 and [Coulot et al., 2007]. Then, a first computation is carried out with an empirical weighting derived from the mean orbit residual WRMS per station and per satellite.

For this computation, we apply weak constraints on station positions and EOPs. From this data processing results, we get estimated station positions which are used for the second computation. Indeed, for this latter, station positions are held fixed to the previous estimated values and, consequently, the only parameters to be computed are EOPs, the common parameters. From this computation, we then get the weekly optimal weights per station and per satellite which can now be used for any SLR data processing.

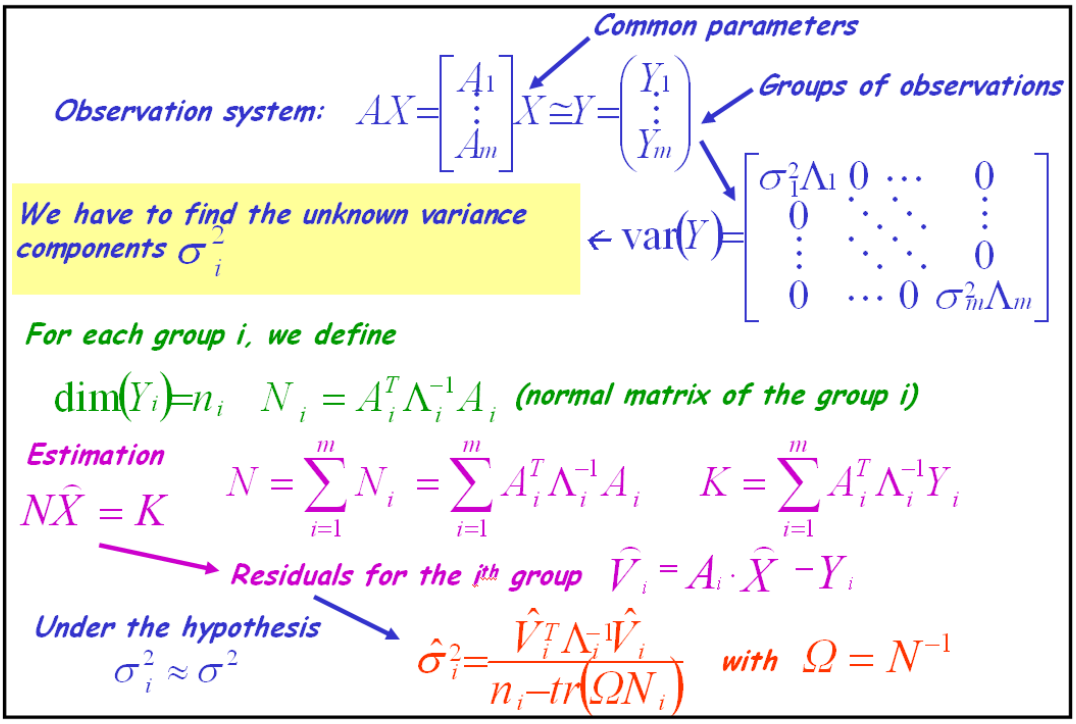

Figure 4. Scheme of the degree of freedom method.

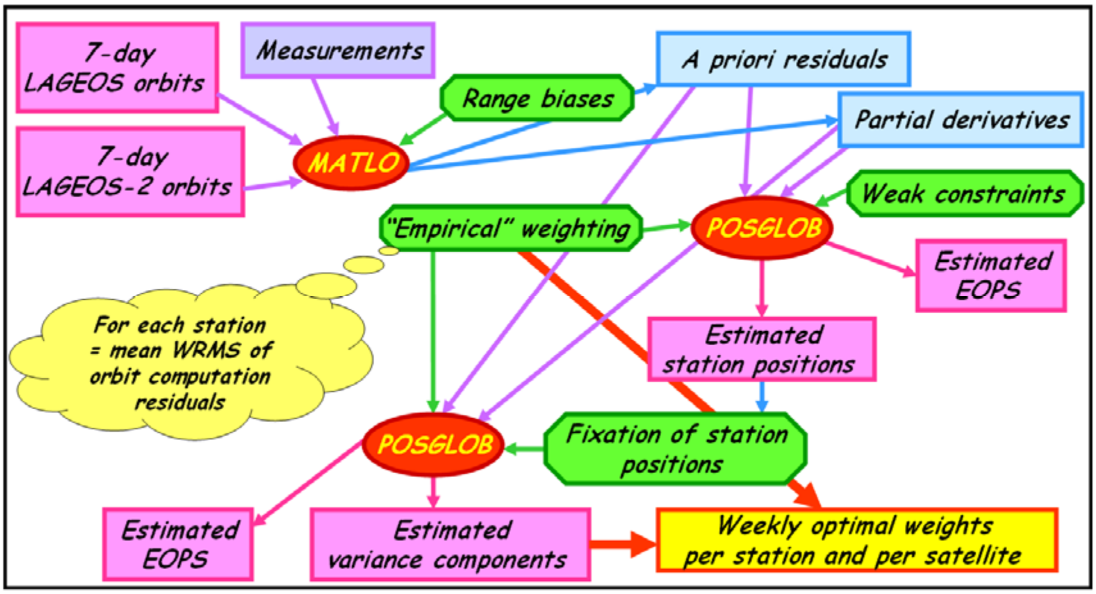

***Figure 5.** Scheme of weekly optimal weight per station and per satellite computation.*

Table 6 provides the mean WRMS values of residuals per station and per satellite computed with the optimal weighting. On the whole, the values are consistent with the a priori knowledge one can have on the SLR network station quality but our approach should be more improved by the use of all the involved parameters to compute the optimal weighting. Indeed, orbital residual error parameters (see next section) are common parameters for measurements per station and we should study the impact of the non common parameters (namely, the station positions) on the results produced by variance component analysis methods. Moreover, these values also evidence the fact that the model used to compute the optimal weighting does not explain the SLR measurements at the millimeter level (the best values are few millimeters). It is certainly mainly due to the fact that the residual orbital errors were not estimated.

Table 6. *Mean WRMS (in cm) values of residuals per station and per satellite computed with the weekly optimal weights derived from the method shown in Fig. 5. For each station, the first (resp. second) column corresponds to the mean WRMS for LAGEOS (resp. LAGEOS-2) satellite. Evidenced stations are present in less than 50 weeks over the 13-year time interval.*

1824	20.5	20.3	7210	1.0	0.9	***7502***	2.3	1.9	7840	0.9	0.9
1831	4.0	3.9	***7231***	5.1	6.2	***7505***	1.6	2.0	7841	1.1	1.1
1863	2.6	2.5	7236	11.4	10.4	7520	1.5	1.3	7843	1.8	1.6
1864	4.0	3.6	7237	2.0	1.9	7548	11.6	6.8	7845	1.0	0.9
1867	30.9	16.2	7249	5.1	4.5	***7597***	2.5	3.1	***7847***	9.9	12.9
1868	9.4	8.1	***7295***	0.9	0.9	***7805***	13.2	15.0	***7848***	2.3	1.9
1873	13.8	14.1	7308	2.0	1.9	***7806***	1.9	1.5	7849	2.3	1.1
1884	2.3	2.1	***7335***	1.0	0.9	7810	1.4	1.4	***7850***	0.7	0.8
1885	8.9	13.0	***7337***	1.0	2.2	7820	2.3	2.4	***7882***	0.5	0.6
1893	3.3	3.3	7339	1.2	0.8	***7821***	2.0	2.9	***7883***	0.5	0.6
1953	9.9	11.5	***7355***	4.3	3.7	7824	2.4	2.3	***7884***	1.2	0.6
7080	1.0	0.8	***7356***	2.8	2.8	7825	1.8	1.9	***7918***	0.9	1.1
7090	1.7	1.4	***7357***	4.9	6.0	***7830***	1.6	1.5	7939	6.9	6.8
7105	0.9	0.8	***7358***	5.0	6.9	***7831***	2.7	2.0	7941	0.9	0.8
7106	7.6	.	7403	1.5	1.1	7832	1.2	1.2	***8833***	2.8	2.7
7109	0.7	0.6	***7404***	4.9	1.8	7835	1.0	0.9	8834	1.4	1.4
7110	0.9	0.8	7405	2.7	2.7	7836	1.0	0.9	7811	1.8	1.6
7122	0.7	0.7	***7410***	0.7	0.6	7837	2.1	2.0			
7124	1.7	1.2	***7411***	0.5	0.6	7838	1.7	1.6			
7130	1.3	1.4	7501	2.2	2.1	7839	0.8	0.8			

2.2. Orbital residual errors

As previously shown in section 1, the LAGEOS satellite orbital arcs may be affected by some residual errors (cf. Fig. 2 and Tab. 5). The integration of Hill's satellite first-order motion differential equations ([Cretaux et al., 1994] and [Coulot, 2005]) provides the empirical form of such orbital residual errors in the RTN frame:

$$\delta R(t) = a_R t\cos(\bar{n}t) + b_R t\sin(\bar{n}t) + c_R \cos(\bar{n}t) + d_R \sin(\bar{n}t) + e_R + f_R t$$
$$\delta T(t) = a_T t\cos(\bar{n}t) + b_T t\sin(\bar{n}t) + c_T \cos(\bar{n}t) + d_T \sin(\bar{n}t) + e_T + f_T t + g_T t^2$$
$$\delta N(t) = a_N t\cos(\bar{n}t) + b_N t\sin(\bar{n}t) + c_N \cos(\bar{n}t) + d_N \sin(\bar{n}t) + e_N$$

The coefficients evidenced in yellow can be estimated. Thus, doing so, we can carry out a kinematic (or semi-dynamic) estimation of the orbital residual errors; see Fig. 6 for examples.

In order to avoid spurious transfers between the terrestrial and the orbital parameters, we should compute all the involved parameters (station positions, EOPs and orbital residual errors) in a same process. But, doing so gives rise to problems. Indeed, it creates supplementary reference system effects [Sillard and Boucher, 2001] on the third translation and on the scale factor of the underlying Terrestrial Reference Frame

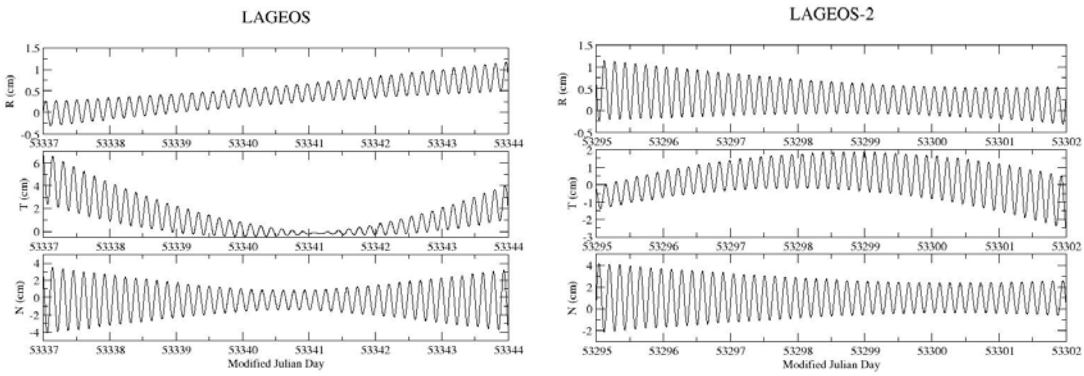

Figure 6. *Examples of orbital residual errors estimated, in cm, for both LAGEOS satellites in the RTN frame.*

(TRF). These parameters are thus damaged and the estimated orbital errors so computed are completely eccentric! Consequently, we have to find a rigorous balance between minimum constraints used to define the weekly TRFs and possible constraints applied on the orbital error coefficients. Furthermore, we have to take into account the physical coupling between the radial and tangential components [Coulot, 2005]. Finally, we have to carry out a sensitivity analysis to determine which coefficients can be optimally computed each week.

2.3. Range biases

Regarding range biases, we have developed a temporal de-correlation method in order to get the most accurate and consistent range bias values (see [Coulot et al., 2007] for more details). Fig. 7 provides an extract of the raw output file provided by this method. We can see that, when they are estimated over long periods, biases per satellite are very coherent. In other cases, the differences are at a few millimeter level.

Station	Validity (CNES J. Dates)		Values obtained for LAGEOS (m)		Values obtained for LAGEOS-2 (m)	
			Bias	+ precision	Bias	+ precision
7835	15340	16544	1.395828547040809E-002	1.158416922784649E-003	1.234432517595224E-002	1.203503017002734E-003
7835	16544	16564	3.529396209728985E-002	1.475235139678072E-002	6.300895927387962E-002	1.990854928253546E-002
7835	16564	16679	3.320933374887905E-002	3.232515758791523E-003	2.891112857748582E-002	3.484324601453627E-003
7835	16679	17413	1.664937112299916E-002	1.430681097343033E-003	1.540554934193744E-002	1.520702451478579E-003
7835	17413	18143	-1.971404917293083E-003	3.328628424293636E-004	-6.434411963804878E-004	3.564485287931656E-004
7835	18143	18873	2.129710997299538E-003	3.360069389674007E-004	3.901844189266957E-003	3.633585927952743E-004
7835	18873	19603	9.131760532044128E-003	4.467022182450903E-004	5.700719983120365E-003	4.759540373836256E-004
7835	19603	20300	7.851262565453349E-003	9.554258819448195E-004	2.842151050185370E-003	1.004394426948261E-003
7845	17521	18305	-4.428851680859768E-003	6.223534556697570E-004	-1.201192209408536E-003	6.893149034575394E-004
7845	18305	19305	8.113087278820413E-003	4.018334706229168E-004	7.840983271450021E-003	4.356245988090018E-004
7845	19305	20035	1.285646431835049E-002	5.459565786674019E-004	1.654336641325428E-002	5.438026909611077E-004
7105	15317	16047	-3.746952854385001E-003	4.944474970946950E-004	-6.864306478378965E-003	5.264430332363305E-004
7105	16047	16777	-1.549706818118012E-002	6.921317933826075E-004	-1.064881791710468E-002	7.505764408214999E-004
7105	16777	16940	-1.105482201853714E-002	5.900205316694585E-004	-7.750502221599706E-003	6.391419698574262E-004
7105	16940	17670	-4.330403233925809E-003	3.766190351993729E-004	-2.921756834785135E-003	4.000337192808937E-004
7105	17670	17877	-8.797353166941418E-003	6.028898715788532E-004	-4.826611780213416E-003	6.371640591200326E-004
7105	17877	17959	-8.741490387424361E-003	1.310118230413150E-003	-1.019322123979381E-002	1.342775122007329E-003
7105	17959	18039	-4.648690714335582E-003	8.785822392202415E-004	-2.871015614103231E-003	9.488477693905651E-004
7105	18039	18666	-5.169111580414628E-003	3.867953242580351E-004	-4.312745700605802E-003	4.185257805300395E-004
7105	18666	18877	-3.472024871091189E-003	5.657230028905748E-004	-2.899406111483316E-003	6.018937033126987E-004
7105	18877	19607	-4.193980900330184E-003	3.817904507077791E-004	-5.491238818875488E-003	4.105820099047128E-004
7105	19607	20337	-6.548560807830205E-003	4.924245800410994E-004	-7.645452603994796E-003	5.332010012145024E-004
7105	20337	20500	-7.040431351813800E-003	9.396523059811853E-004	-6.322205532930170E-003	1.035964446496983E-003

Figure 7. *Examples of range bias values (m) per station and per satellite computed with the temporal de-correlation method [Coulot et al., 2007]. CNES JD=MJD-33 282.*

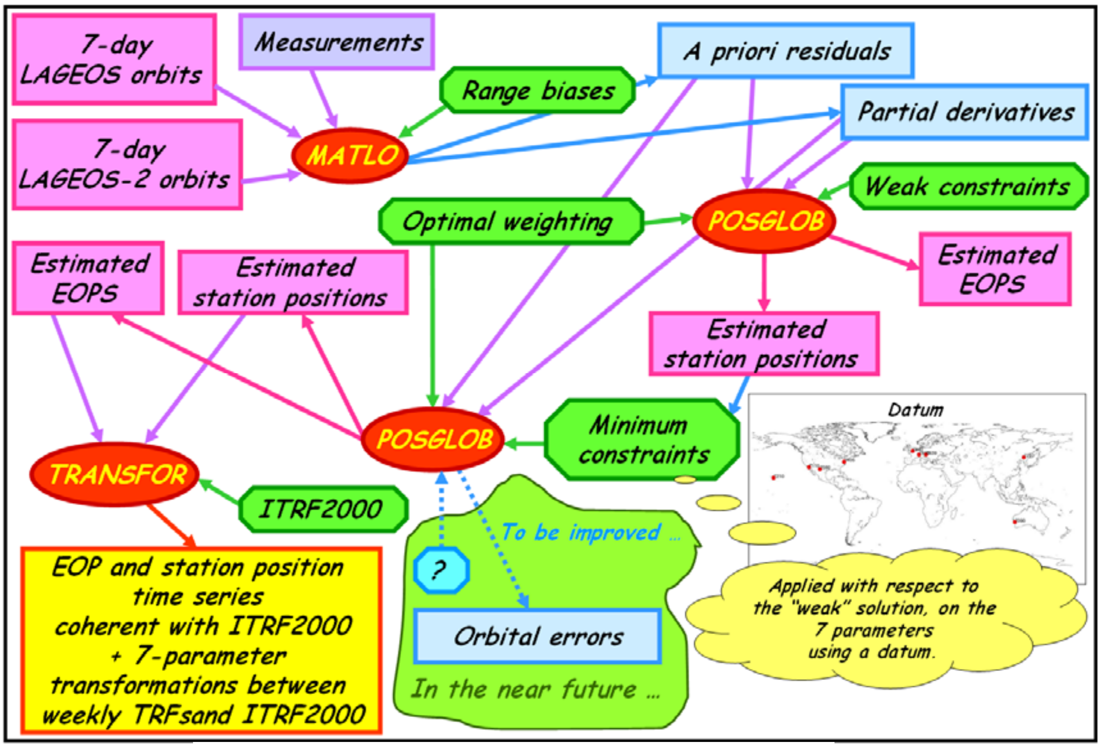

Figure 8. *Time series computation method scheme.*

3. Time series computation

3.1. Method

Fig. 8 shows the global method scheme. For the time series computation, the range bias values computed with our new method as well as our optimal weights are applied. For this first "long period" data processing carried out with MATLO/POSGLOB software, no orbital residual error is estimated nor applied.

3.2 Results

Fig. 9 shows the results produced with TRANSFOR software (cf. Fig. 8) for the three translation parameters. We have carried out frequency analyses of these time series. These analyses have been carried out with FAMOUS (Frequency Analysis Mapping On Unusual Sampling) software developed by F. Mignard (OCA, France) in the framework of the GAIA project [Mignard, 2004]. The TX (resp. TY) time series exhibit a 2.9 mm (resp. 3.2 mm)-amplitude annual signal and the TZ time series exhibit a 2.4 mm-amplitude annual signal as well as a 1.7 mm-amplitude semi-annual signal. Moreover, the scale factor time series are shown in [Coulot et al., 2007], Fig. 10. They exhibit a 2.6 mm-amplitude annual signal. This annual signal may be an artifact due to the SLR network geometry and the fact that the atmospheric loading effects have not been considered in the a priori modeling used for station positions (see next results for these station positions).

Regarding EOPs, the results are shown on Fig. 10. The weighted biases are respectively -119 and 7 µas for Xp and Yp and the WRMS are respectively 299 and 256 µas for Xp and Yp. Moreover, the opposite drifts detected between 2000.0 and 2006.0 certainly come from some network effects.

The station position time series are estimated with respect to the ITRF2000 mean position corrected for plate tectonics (ITRF2000 velocities), Earth solid tides, pole tide and oceanic loading effects in agreement with the IERS conventions [McCarthy

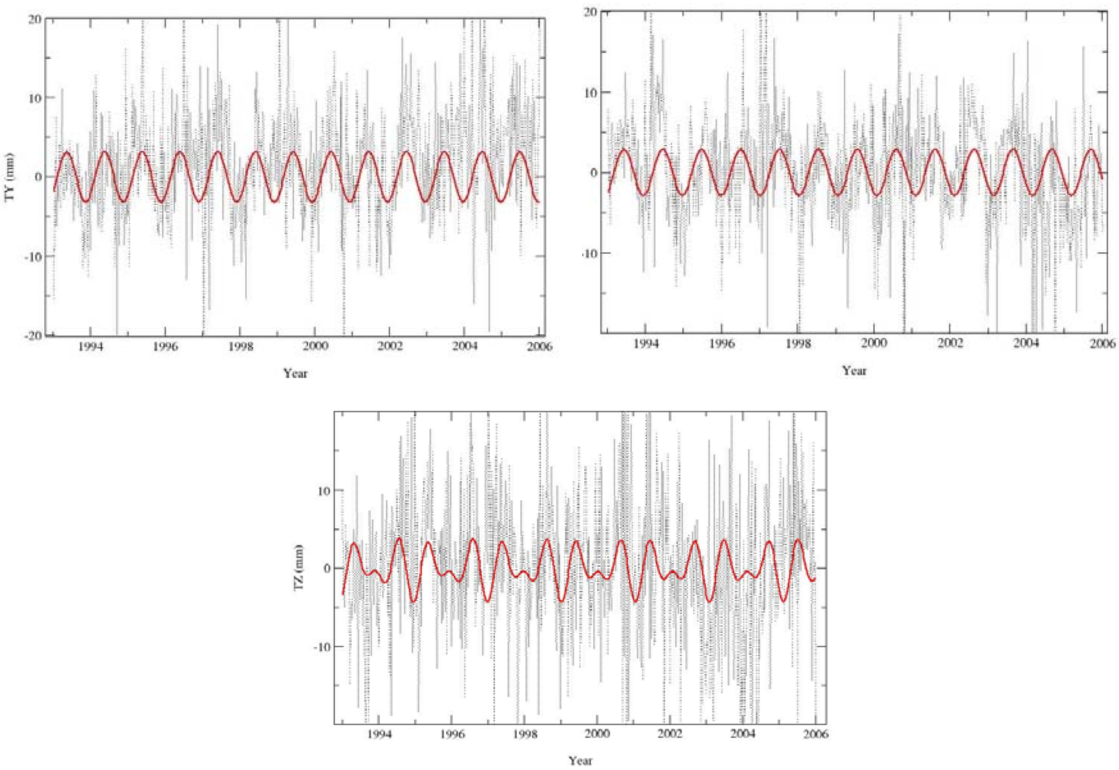

Figure 9. *Weekly translation parameter time series (mm) between weekly SLR TRFs and ITRF2000. Red curves correspond to the periodic signals detected and estimated with FAMOUS software.*

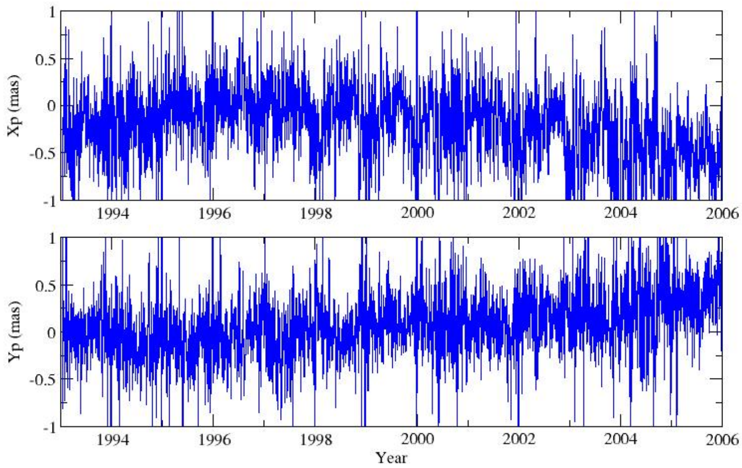

Figure 10. *Daily EOP time series (mas) computed with respect to the EOPC04 time series.*

and Petit, 2004]. These time series must consequently evidence the atmospheric and hydrologic loading effects.

Fig 11 shows 7839 and 7840 SLR station position time series in ITRF2000. Annual and semi-annual signals with amplitudes between 5 mm and 1 cm are detected by FAMOUS software in such Up component time series for some stations. These annual signals may be linked to the previously mentioned loading effects.

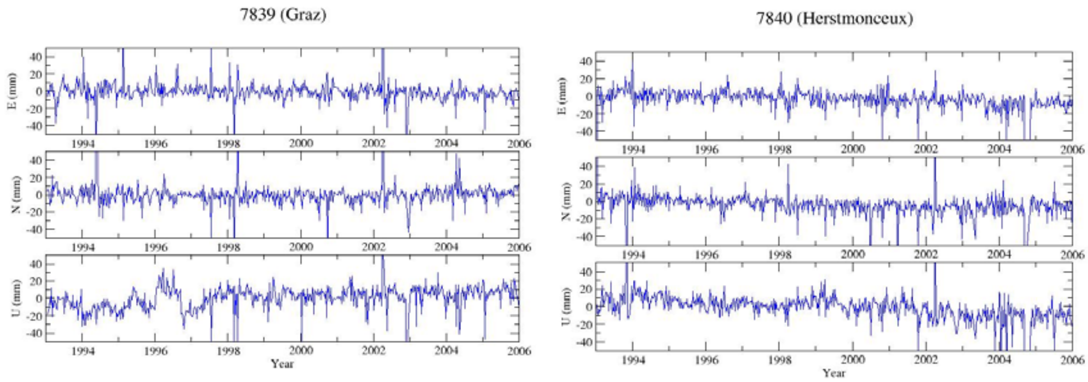

Figure 11. *Examples of station position time series computed (in mm) in the ENU local frame in ITRF2000. On the left: Graz, 7839. On the right: Herstmonceux, 7840.*

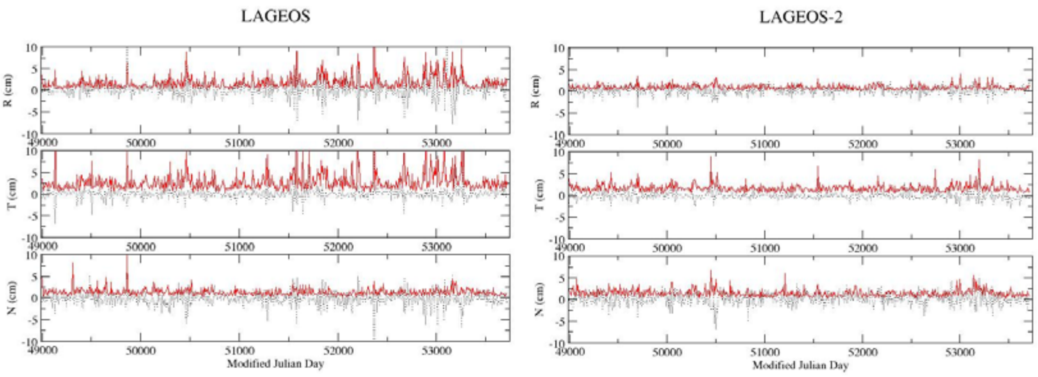

Figure 12. *Empirical orbital errors (biases - in black - and RMS values - in red -, in cm) in the RTN frame computed with our semi-dynamic approach for both LAGEOS satellites (LAGEOS on the left and LAGEOS-2 on the right).*

Table 7. *Statistics of the values shown on Fig. 12.*

Satellite	**R (cm)**	**T (cm)**	**N (cm)**	**.**
LAGEOS	0.38 1.71	0.06 2.73	-0.13 1.32	Mean of means Mean of RMS
LAGEOS-2	0.31 0.90	-0.11 1.65	0.20 1.46	. .

Finally, although our global method (cf. Fig. 8) does not provide any orbital error estimate, we have tested our semi-dynamic approach by keeping station positions and EOPs fixed. Almost all effects are included in the a priori modeling then used for station positions: plate tectonics, solid Earth tides, pole tide, and oceanic and atmospheric loading effects (European Center for Medium-range Weather Forecasts - ECMWF, http://www.ecmwf.int/- pressure fields were used to derive the atmospheric loading effect models) as well as the range biases provided by the temporal de-correlation method. Fig. 12 shows the bias and the RMS values of the empirical orbital errors so computed in RTN frame for both satellites. Tab. 7 provides the mean values of these error bias and RMS values. These values are coherent with the 2-day LAGEOS overlaps (cf. Fig. 2 and Tab. 5).

4. Conclusions and prospects

Our time series estimation method should be operational soon. To do so, we still have to:

- finalize our method regarding orbital errors;
- use all available common parameters to get optimal weekly weightings;
- go further with our temporal de-correlation approach for range biases [Coulot et al., 2007].

New computations should be carried out with ITRF2005 and the improved EOPC04 time series. And, in the near future, we plan to:

- carry out computations with atmospheric loading effect models in the a priori modeling for station positions to quantify their impact;
- use other satellites and study the impact on the involved TRFs.

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